GRAPHS AND NETWORKS FOR YEARS 7 TO 10

Reasons for and ways of using digital technologies to teach algebra and the standard normal curve

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Introduction

Knowledge of mathematics and science is widely regarded as being an essential source of innovation and productivity in a nation's economy. But the declining achievement standards of Australian high school mathematics students was much in the news late last year (ABC News, 2013) as the latest OECD rankings showed that our 15-year-olds rank equal 8th of all nations for science and merely equal 17th in mathematics. The results show Australian students are slipping in mathematics performance by about half a year of schooling compared to 10 years ago and the decline was stronger in girls than boys.

However, if you want to interest students in mathematics (and science) you must engage them in the lower forms of high school or even earlier (Fisher, 2012). So, teachers should always consider a topic's ability to interest students in the early years of instruction in high school and its topicality. Networks have come into prominence recently with an episode of *Four Corners* on ABC (Gould et al., 2013) and the popular film, *The Social Network* (Fincher et al., 2010) so discussion of the program or the film and even a viewing of one or two scenes, along with discussion of aspects of social networking such as Facebook and Twitter, are a good way to introduce the topic.

Graphs are representations of networks and it makes sense to teach them together. Also, teaching them together can be an interesting example of applied mathematics and how mathematicians and scientists work together collaboratively when applying mathematics and developing a new scientific theory (Jacques, 2008). It is a good way of teaching mathematics *in context*. The topic is also relevant to the new Australian curriculum re graphs and graph theory (ACARA, 2011) and it provides an excellent opportunity to 'embed digital technologies so that they are not seen as optional tools' (ACARA, 2009, p. 12).

Introducing the topic: Students' interests

This article shows how graphs and networks can be introduced in Years 7 to 10 in a variety of ways depending on: students' interests, their preoccupation with the Internet, game playing (Reynolds, 2009) and social networking sites such as Facebook, and teachers' co-operation with physics, science, or even, drama teachers (Guare, 1990). It shows how a good documentary on the subject with an intriguing title—*How Kevin Bacon Cured Cancer*—can also be extremely useful (Jacques, 2008).

Curriculum considerations Years 7–10

As is noted by the Australian Curriculum, Assessment and Reporting Authority [ACARA], students at this level need to: "develop more complex mathematical ideas … to motivate them during these years, students need an understanding of the *connections between the mathematics concepts and their application in their world* in contexts that are directly related to *topics of relevance and interest to them*" (ACARA, 2011, p. 8; author's emphasis).

As for teaching the standard normal curve in Year 10, ACARA (2014) advocates teaching algebra, probability and 'a curve of best fit' in Year 10 (ibid, p. 41) and this seems to be not a big step from teaching the normal standard curve. This paper argues that with the digital technologies currently and ever increasingly available and the structured, student-interests-centred, long-term introduction of the topic suggested in this paper, it is possible to teach the standard curve in Year 10, especially with the cooperation of teachers from other disciplines.

Mathematics today—according to Wolfram

An Internet site such as Wolfram's (2010) admirably shows the advantages of using digital technology in the classroom to teach mathematics. Any student capable of using the Internet or *Excel* can make, for example, spreadsheets and standard normal curves with these technologies. Understanding the standard normal curve is a good foundation for learning about power-law graphs in Years 11 and 12 (Padula, 2012).

But what about the future of mathematics? What does it hold?

The future of mathematics

According to experts heard on the ABC (Smith & Davies, 2012), almost everything we teach our students up to the age of 14 or 15 is from the 9th and 10th centuries. We are still teaching calculus 300 years after it was discovered and virtually nothing learnt in mathematics since the eighteenth century. Yet the mathematics we need to use in the world today has changed. It was important to do arithmetic, now we use calculators. Today we have to be good at *reasoning with numbers*. We use spreadsheets a lot and computers do the calculations, but you have to describe the spreadsheet, which is *algebra*. A spreadsheet such as can be drawn in *Excel* offers an environment in which algebraic ideas from initial concepts of pro-numerals and variable, through to simplification of expressions to graphs and functions can be developed (Horne, 1995).

The mathematics behind networks science (Padula, 2012) is an excellent example of how scientists *apply* mathematics to problems facing the world today. So, what is a graph and why are networks important?

What is a graph?

As previously stated, in mathematics a network is represented by a graph. A graph is a pair of sets, where P is a set of N nodes (or vertices or points) P_1, P_2, … P_N and E is a set of edges (or links or lines) that connect two elements of P. Graphs are usually represented as a set of dots, each corresponding to a node, two of these dots being joined by a line if the corresponding nodes are connected (see Figure 1 from Albert & Barabási, 2002).

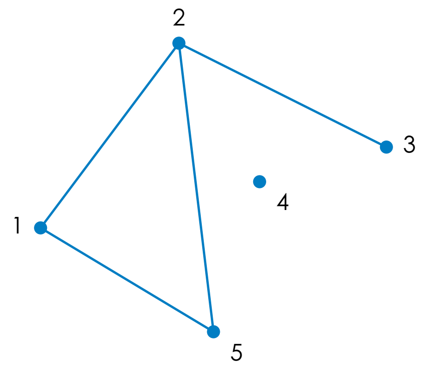

Figure 1. Illustration of a graph with N = 5 nodes and n = 4 edges (Albert & Barabási, 2002, p. 9). Figure reproduced with permission from Albert, R. & Barabási, A.-L., (2002) Reviews of Modern Physics, 74, p. 54. Copyright by the American Physical Society.

As can be seen above, the set of nodes is $P = \{1, 2, 3, 4, 5\}$ and the edge or link set is $E = \{\{1, 2\}, \{1, 5\}, \{2, 3\}, \{2, 5\}\}$.

Importance of networks

Networks are important because, as Barabási (2002, p. 16) has remarked: "…computers linked by phone lines, molecules in our body linked by biochemical reactions, companies and consumers linked by trade, nerve cells connected by axons, islands connected by bridges are all examples of graphs."

Whatever the identity and nature of the nodes and links, for a mathematician they form the same animal: a *graph* or a *network*.

Graph theory

After Euler solved the Bridges of Konigsberg problem in 1735 and founded graph theory, the theory of *random* graphs was proposed by Paul Erdös and Alfréd Rényi (Erdös & Rényi, 1959, 1960) after Erdös found that it was advantageous to use *probability* methods when tackling problems in graph theory. In their definitive first article on random graphs, Erdös and Rényi define a random graph as N labelled nodes connected by n edges or links which are chosen randomly from the

$$\frac{N(N-1)}{2}$$

possible edges (Erdös & Rényi, 1959). In total there are

$$C^{n}_{\frac{N(N-1)}{2}}$$

graphs with N nodes and n edges, forming a probability domain in which every realisation is equiprobable or equally probable (ibid).

The greatest discovery of Erdös and Rényi was that many important properties of random graphs appear *suddenly*, but their work on graph theory was not improved upon until the development of computers made the study of very large networks possible (Barabási, 2002). Late in the twentieth century, in 1998, further developments ensued with the recognition by two researchers of the existence of 'small worlds'.

Synchronicity and 'small worlds'

Steven Strogatz, an applied-mathematics professor at Cornell University, was intrigued by synchronicity in nature. Strogatz and Australian Duncan Watts (1998), then a graduate physics student, studied synchronicity in snowy tree crickets, how they can make their sound in unison when far apart. Watts' father reminded him of "six degrees of separation" (Guare, 1990), the idea that everybody is just six steps or links from anyone else on earth. (The idea is: if you are one step or link away from each person you know and two steps away from each person who is known by one of the people you know, then everyone is an average of six steps away from every other person of the seven billion people on Earth.) Mathematically, 'points' (people, insects, etc., or mathematically speaking: vertices) are connected by 'lines' (links or edges) and the question was asked, do they make a 'small world'?

Six degrees of separation

Needing to study a ready-made network, the researchers thought of Hollywood and the game, *The Oracle of Bacon*, devised by Professor Brett Tjaden (Reynolds, 1999). With this game, every Hollywood actor is, at most, six connections away from every other actor, including Kevin Bacon, and participants can test their knowledge of films by nominating an actor and estimating the distance (in connections or links) between the actor and any other actor. They also studied an American power grid, and, since each nerve in the brain is just a neuron away from other neurons, the nervous system of a worm, a nematode, *Caenorhabditis elegans*, or *C.elegans*, which had already been mapped (Watts & Strogatz, 1998).

A small-world model

In the Watts and Strogatz small-world model, N vertices form a one-dimensional lattice, each vertex being connected to its nearest and next-nearest neighbours. With probability p, each edge is reconnected to a vertex chosen randomly, with the proviso that no two vertices can have more than one connection, and no vertex can have a connection with itself (Barabási, Albert & Jeong, 1999). The long-range links generated in this process decrease the distance between the vertices, leading to a small-world phenomenon, often referred to as six degrees of separation (Guare, 1990; Watts & Strogatz, 1998; Singh, 2013).

The Watts–Strogatz model begins with a one-dimensional lattice of N vertices with links with the nearest and next-nearest neighbours (in general, the algorithm can include neighbours up to an order n, such that the coordination number of a vertex is $z = 2n$).

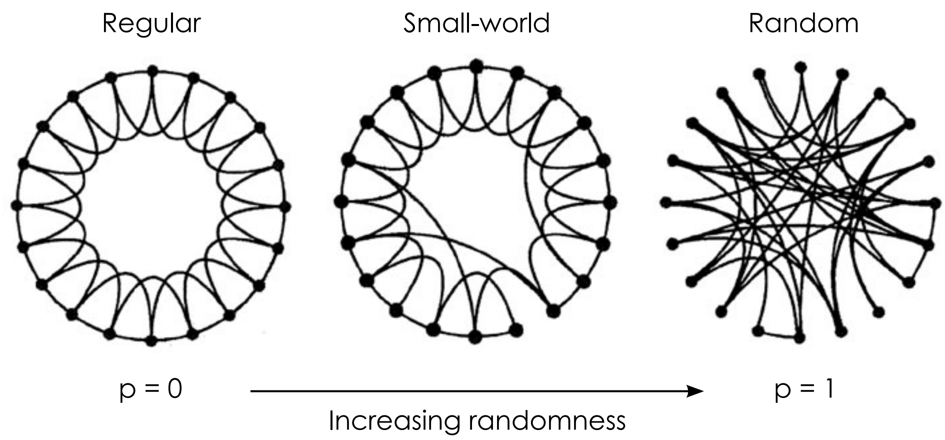

Figure 2. The Watts, Strogatz (1998) model of the 'small-world' phenomenon (Watts & Strogatz, 1998, p. 441). Reprinted by permission from MacMillan Publishers Ltd: Nature, Watts, D. J.& Strogatz, S.H. (1998), Collective dynamics of 'small world' networks, 393, 440–442, © 1998.

Figure 2 shows the random rewiring procedure of the Watts-Strogatz model which interpolates between a regular ring lattice and a random network without altering the number of nodes or edges. It starts with $N = 20$ nodes, each connected to its four nearest neighbours. For $p = 0$ the original ring is unchanged; as p increases the network becomes increasingly irregular until, for $p = 1$, all edges are rewired randomly (ibid).

It is important to point out that this model has its roots in social systems where most people are friends with their immediate neighbours, people living in the same street, colleagues, people that their friends introduce them to. On the other hand, everybody has one or two friends who are a long way away—people in other countries, old friends or acquaintances who are represented by the long-range edges obtained by rewiring the Watts-Strogatz model. For example, in the Jacques (2008) documentary, *How Kevin Bacon Cured Cancer*, a woman living in a village in Kenya passes a parcel onto an aunt from a town who passes it on to an intermediary in America who can contact someone who knows someone who knows a scientist, the desired recipient, in all with six links.

Comparison of the Erdös–Rényi and Watts–Strogatz models

Barabási, Albert & Jeong (1999) compare the Erdös–Rényi (ER), or random model, with the Watts–Strogatz (WS), or small-world model, focussing on the connectivity distributions (see Figure 3).

The data for Figure 3 is as follows:

(a) $P(k)$ in the Erdös–Rényi model for $N = 10\,000$ and $p_{ER} = 0.0006$ (circles), $p_{ER} = 0.001$ (squares) and $p_{ER} = 0.0015$ (diamonds).

(b) $P(k)$ in the Watts-Strogatz model for $N = 10\,000$, $<k> = 6$ and three rewiring probabilities $p_{WS} = 0$ (circle, corresponding to the delta-function $\delta(k - 6)$), $p_{WS} = 0.1$ (squares) and $p_{WS} = 0.3$ (diamonds).

As can be seen from Figure 3, the peaked distributions of these networks resemble normal standard curves, sometimes referred to by sociologists as 'bell' curves (Weisstein, 2013) that occur most often in quantities in nature with topics as diverse as: heights in a population, IQ range, test scores, the width of stripes on a zebra, the measure of LDL cholesterol in adults, most measurement errors and the velocity of molecules in a gas (Barabási, 2002; Kyd, 2012). The mean of a large number of random variables independently

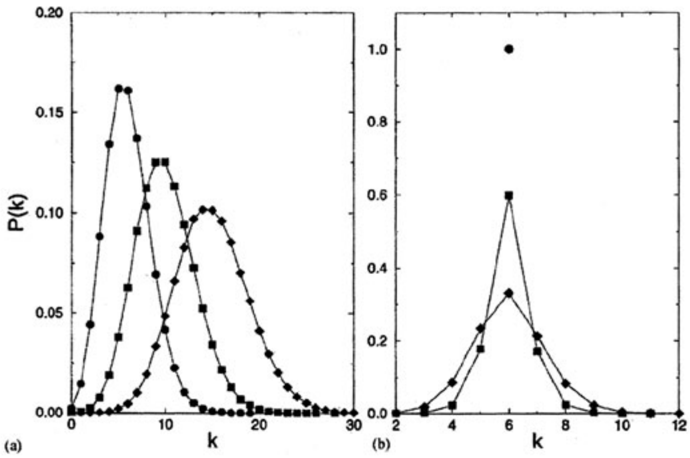

Figure 3. Connectivity distributions for the Erdös–Rényi and Watts-Strogatz models (Barabási, Albert & Jeong, 1999, p. 177). Reprinted from Physica A, 272 (1-2), Barabási, A.-L., Albert, R. & Jeong, H., Mean-field theory for scale-free random networks, 173-187, copyright (1999), with permission from Elsevier.

drawn from the same distribution is distributed approximately normally according to the *central limit theorem* (Kyd, 2012).

The distribution characteristics of items manufactured according to some fixed standard (such as component length) fall within normal distribution. The more data items there are, the closer the distribution is to normal standard distribution (Casio Electronics, 2008). As can be observed, these curves have exponentially decaying tails.

The mathematician Gauss defined the standard normal distribution as

$$f(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}$$

The probability distribution function of a variate x is expressed in the following equation in Wolfram's *MathWorld* (Weisstein 2013, p. 1)

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

where μ is the mean and the parameter σ is its standard deviation and its variance is σ^2.

Classroom applications for Years 7–10

Students can search for 'The Bridges of Konigsberg' problem on the Internet and ideally, complete individual tasks based on it in a school mathematics-task centre (Loy, 1999).

After watching the documentary *How Kevin Bacon Cured Cancer* (Jacques, 2008), students can play *The Oracle of Bacon* game (Reynolds, 2009), matching pairs of actors to see how many links (or edges) there are between them. If they choose a relatively obscure supporting actor or actors there may be more links in the 'small world' of Hollywood actors.

Students can make standard normal curves in the classroom. They can assemble their own data by measuring say, the heights of the students in the class or several classes, or if they are physics students and the physics teachers in the school are willing to work with the mathematics teachers they may use some appropriate physics data. The science curriculum contains many references to the importance of mathematics to physics learning. It details the relevant mathematical skills, including graphing (ACARA, 2009, p. 11).

Once they have assembled the data they can find the average, or mean, and the standard deviation (from the mean). With this data they can make a standard curve on a computer with *Excel*. For pre- or post-teaching teachers may also choose to use an instruction video from YouTube (Lane, 2014).

As for finding internet sites with instructions which generate the standard normal curve it is not recommended that students follow instructions that are too long and involved. It may be stating the obvious, but whether students are using a PC or a Mac the instructions they employ must match the version of *Excel* currently on their computers. Please note the following Internet sites which have 7 and 12 steps respectively: Ellen (2013), Gronot (2013) for *Excel 2007* and Ellen (2012), Kyd (2012).

Or, if you agree with the school of thought that claims *Excel* currently does not have a simple way of creating a bell curve out of data, a bell curve-like chart can be made by following the few (12) instructions at the Santa Clara University (2013) site. The data on site, test results of 20 students, is supplied, as are instructions for *Excel*: 2003, 2004, 2007, 2008 for both PCs and Macs, see Figure 4. Alternatively and very simply, these values and frequencies can be used for the *x*, *y* coordinates when making the graph in *Excel*.

Student	Score	Description	Value	Frequency
Student 1	89	<=70	70	1
Student 2	83	>70 and <=75	75	1
Student 3	90	>75 and <=80	80	1
Student 4	86	>80 and <=85	85	4
Student 5	76	>85 and <=90	90	6
Student 6	87	>90 and <=95	95	5
Student 7	98	>95 and <=100	100	2
Student 8	84			
Student 9	83			
Student 10	100			
Student 11	89			
Student 12	70			
Student 13	94			
Student 14	72			
Student 15	92			
Student 16	90			
Student 17	91			
Student 18	92			
Student 19	82			
Student 20	94			

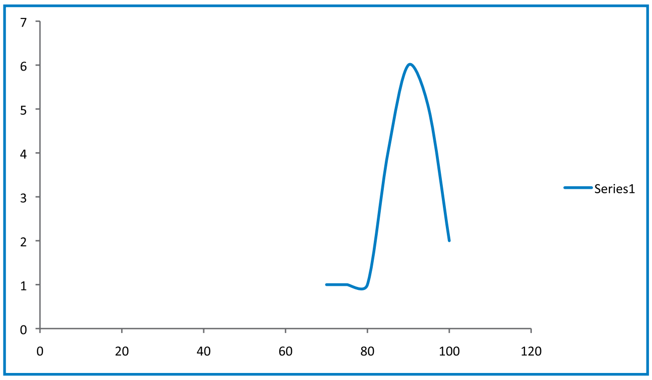

Figure 4. A bell curve-like chart generated in Excel from the data provided on the Santa Clara University (2013, p. 2) site (with frequencies added).

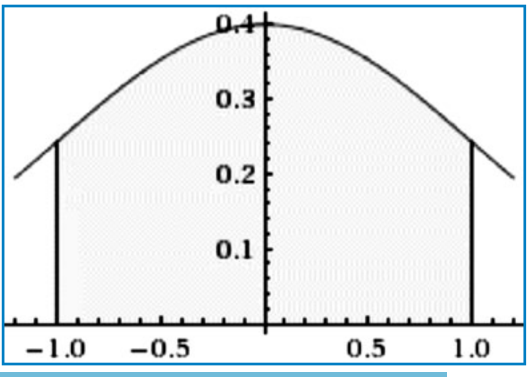

Figure 5. An example of a standard normal curve generated on the Wolfram/Alpha Widgets (2013) site.

Also, the Wolfram/Alpha Widgets (2013) Internet site ably demonstrates the concept of the standard normal curve and can be fun for students at this level. The total area under the curve is equal to one. This function is symmetric around $x = 0$ and has upper and lower boundaries at +1 and –1. After watching the curve form, students can insert their own values and take note of the differences with varying upper and lower boundaries.

Conclusion

Wolfram's (2010) views encourage teachers to 'embed digital technologies' as recommended in the Australian curriculum (ACARA, 2009). The standard normal curve is an ideal topic to do this, in that it is extremely well illustrated by technologies such as the computer program *Excel* and many internet sites such as Wolfram's (2013) and Lane's (2014). Today's highly technologically aware students will enjoy using them as they learn to increase their mathematical and technical skills.

With the documentary by Jacques (2008) students will see how mathematicians and scientists, inspired by events as puzzling and diverse as crickets chirping in unison and actors collaborating in movies have worked together to develop an extremely useful mathematical theory with many very important applications such as: medical research, disease prevention, warfare, the capture of terrorists, and conduct effecting business and financial markets. It will encourage them to study mathematics further or even consider mathematics as a career option.

So mathematics teachers, alone, in a team, or in collaboration with physics, science, or English teachers, have the choice of introducing the topic with: an episode of the television program *Four Corners* (Gould et al., 2013), the computer game, *The Oracle of Bacon* (Reynolds, 2009); Bridges of Konigsberg-problem activities (Loy, 1999); the play, *Six Degrees of Separation* (Guare, 1990); the film of the same name, produced and directed by Fred Schepisi (Kidney, Milcham & Schepisi, 1993); the film *The Social Network* (Fincher et al., 2010) or, simply, by allowing students to view the extremely interesting and informative documentary (Jacques, 2008) *How Kevin Bacon Cured Cancer*.

Students learning algebra can have difficulties seeing its relevance. The concepts and context need to be presented in a manner enabling them to attach meaning to the symbolic language of algebra (Horne, 1995). So, in terms of the curriculum, why not discuss aspects of social media and popular culture such as: Facebook, Twitter, YouTube, recent documentaries and films, a play, or an episode of a television program as an introduction to graphs and networks in secondary school? And at the same time demonstrate the importance of mathematics in today's world—with modern technologies and sites easily available on the Internet, thereby laying the foundation for power-law graphs in the final two years of high school, or early years of university (Padula, 2012).

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